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Geometrical Relationship Between Differentiation and Integration

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Abstract

The topics namely 'Differentiation' and 'Integration' play a vital role at lower and higher levels of education in the field of Mathematics. In other words these topics provide a strong foothold for acquiring higher knowledge of Mathematics. Usually we observe that students of Mathematics go on differentiating and integrating the problems without knowing what actually they are doing. The article is completely focused on the issue how a student of mathematics can be inculcated to understand the physical and geometrical relationship of the topics under discussion. The description given in this article shall enable even a common man having a mediocre knowledge about mathematics to know how to play with differentiation and integration particularly when dealing with geometrical figures such as curves, surfaces etc.

Keywords: Differentiation, Integration, Relationship, Knowledge, Circle, Cylinder, Volume, Geometrical, Physical, Application, Newton, Leibnitz, Lebesque.

Introduction

The subject of differential calculus is a development of the important notation of instantaneous rate of change which is itself a limit idea, as distinguished from the average rate of change and as such it finds application to all those branches of knowledge which deal with the same. Thus it is applied to geometry, Mechanics and other branches of theoretical Physics and also to Social Sciences such as Economics and Psychology. The idea of differential calculus was first conceived by Newton (1643-1727) and Leibnitz (1646 – 1716). Next, the integration in elementary treatment, the subject of integration is treated from the point of view of the inverse of differentiation so that the function g is called an integral of a given function f(x) if $g^{/}$(x) = f(x), for all x belongs to the domain of function f(x). Historically, however, the subject arose in connection with determination of the areas of plan regions and was based on the notion of the limit of a type of sum. In fact the integral calculus has had its origin in this process of summation and the words 'to integrate' literally means 'to give the sum'. In elementary works the reference to integration from summation point of view is always associated with intuitively perceived geometrical concepts. No doubt rigorous arithmetic treatments free from geometrical notions of integration was first given by Riemann (1782-1867), while Cauchy (1782-1857)earlier had confined the scope of his theory of integration to continuous functions only. The most noteworthy of these is the theory of integration by Lebesque (1895-1941) in 1902.

Objectives

1. To know what is differentiation and integration geometrically or physically.
2. How these topics are co-related.
3. How a student of mathematics can be made familiar with the originality of the topics.
4. How we can deal with the figures in our day to day life in the context of the said article.
5. To provide good understanding of the topics to the students of mathematics in particular and to a common man in general.

Now, I will explain my view point with the help of following illustrations:
Consider a circle of radius r and let A represents the area of the circle i.e.
$A = \pi r^2$sq. ft. (I) $\longrightarrow$
Diff. = n (I) on b/s w.r.t.r we get:
$\frac{dA}{dr}$= 2 πr (II)
Which means the rate of change of area of a circle is its circumference. If we analyze it in physical sense, it means we carve out circumferences 2 π r

from the area of circle $A = \pi r^2$ regarding r as in decreasing order of size of radius of the circle. If broadly speaking, suppose r > $r_1 > r_2 > r_3 >$ ……

Therefore, we have A – 2πr = A_1 (where A_1 is the remaining area of circle after removing the circumfrance 2πr)

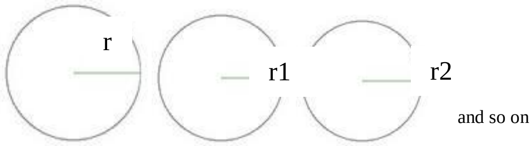
A_1 - 2πr_1 = A_2 (where A_2 is the remaining area after removing the circumference 2πr_1 from area A_1)

A_2 - 2πr_2 = A_3 (where A_3 is the remaining area after removing the circumference 2πr_2 from area A_2)

And so on.

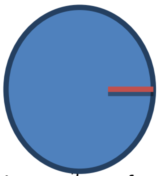
From the above it follows that we have carved out circumferences 2πr, 2πr_1, 2πr_2, 2πr_3, …… from the area of the circle $A = \pi r^2$

Which geometrically means:



2πr2πr_1 2πr_2

Have been carved out from the circle



$A = \pi r^2$ sq. ft

Now, if we add all such carved out circumfrances according to their decreasing order in geometerical sense such as 2πr + 2πr_1 +2πr_2 ...... = $A = \pi r^2$ sqft

This process is same as integrating = n (II) on b/s w.r.t r

i.e.

$\int \frac{dA}{dr}$ dr = $\int$ 2πrdr

=>$\int dA$ = $\left(\frac{2\pi r^2}{2}\right)$ = πr^2

=>A = πr^2

This pattern of interpreting geometrically the integration as the inverse of differentiation is very obvious and easy to understand the concepts of differentiation and integration.

Consider the volume of a cylinder i.e. V = πr^2hcu. Units, Where r is the radius of the cylinder and h is the height of the cylinder

Since V = πr^2h cu units →(I)

Diff. on b/s w.r.t h, we get

$\frac{dV}{dh}$= πr^2→(II)

i.e. rate of change of V w.r.t.h. is a circle of area πr^2

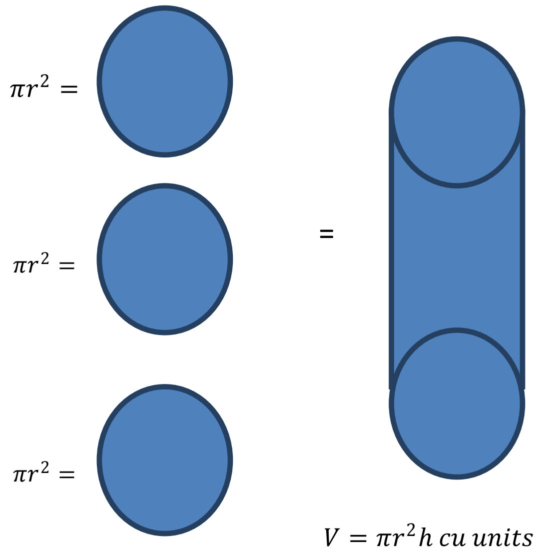
Again if we consider its physical or geometrical significance, it means we cut the height of the cylinder or log of wood of volume V = πr^2hcu. units into many circular slices of the same area πr^2regarding h as parameter. And, if we add all such circles of area πr^2 we can again get the cylinder of volume V = πr^2h cu. Units. This is same as if we integrate = n (II) on b / s w.r.t.hi.e

$\int \frac{dV}{dh} dh$ = $\int \pi r^2$ dh

=>$\int dV$ = πr^2h

=> V = πr^2h cu. units

Which geometrically means



and son on

Hence, geometrically or physically differentiating means breaking of cylinder of volume V = πr^2h into different circles of area $\pi r^2$and integration means adding all such circles of area Пr^{2}to get the required cylinder of volume V = πr^2h.

Again if we diff. = n (I)w.r.t.r on b/s we get

$\frac{dv}{dr}$= 2πrh →(III)

Which means rate of change of volume of cylinder w.r.t.r is its curved surface area. But physically discussing the significance of differentiation and integration, in the present case we can carve out many cylindrical surfaces or cylindrical layers from the log of wood or cylinder of volume V = Пr^{2}h cu. units. And again if we fix all such cylindrical layers or surfaces, we can get the required volume of cylinder or log of wood i.e. V = πr^2h cu. Units such as

and so onV = πr^2h

The above method is same as integrating = n (III) on b/sw.r.t r

i.e.

$\int \frac{dV}{dr} dr$ = $\int$ 2πrh dr

$\int dV$ = $\frac{2\pi r^2 h}{2}$ = πr^2h

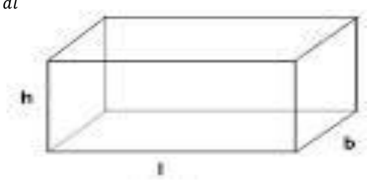
=>V = πr^2h cu units

Consider V the volume of a cuboid having length equal to l, breadth b and height as h

V = lbh cub units → I

Diff w.r.t l we get

$\frac{dv}{dl}$ = bh → II



i.e. rate of change of volume w.r.t length l is the vertical rectangle having area bh. It physically means we have broken the coboid into many vertical rectangles of area b x h

If we add all such vertical rectangles we can again get the cuboid

V = lbh, as we have done in above cases geometrically.

The above process is same as integrating = n (II) w.r.t l on b/s i.e

$\int \frac{dV}{dl} dl$ = $\int$ bh dl dr

$\int dv$ = lbh

⇨ V = lbh Cu units

In this way we can make clear the concept of differentiation and integration to our students in particular and to a common man in general.

This approach is based on my intrinsic point of view as per the physical or geometrical significance of the two topics "Differentiation and Integration" is concerned. I can give many more such examples, but the above mentioned are sufficient to understand the physical relationship between differentiation and integration. The way I have dealt with some geometrical figures mentioned above is not a general one, for that, reasons can be given in the next article.

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